An Unconditionally Stable Scheme for the Finite-Difference Time-Domain Method

Young-Seek Chung, *Member, IEEE*, Tapan K. Sarkar, *Fellow, IEEE*, Baek Ho Jung, and Magdalena Salazar-Palma, *Senior Member, IEEE*

***Abstract*—In this work, we propose a numerical method to obtain an unconditionally stable solution for the finite-difference time-domain (FDTD) method for the TE_z case. This new method does not utilize the customary explicit leapfrog time scheme of the conventional FDTD method. Instead we solve the time-domain Maxwell's equations by expressing the transient behaviors in terms of weighted Laguerre polynomials. By using these orthonormal basis functions for the temporal variation, the time derivatives can be handled analytically, which results in an implicit relation. In this way, the time variable is eliminated from the computations. By introducing the Galerkin temporal testing procedure, the marching-on in time method is replaced by a recursive relation between the different orders of the weighted Laguerre polynomials if the input waveform is of arbitrary shape. Since the weighted Laguerre polynomials converge to zero as time progresses, the electric and magnetic fields when expanded in a series of weighted Laguerre polynomials also converge to zero. The other novelty of this approach is that, through the use of the entire domain-weighted Laguerre polynomials for the expansion of the temporal variation of the fields, the spatial and the temporal variables can be separated. To verify the accuracy and the efficiency of the proposed method, we compare the results of the conventional FDTD method with the proposed method.**

***Index Terms*—Finite difference time domain (FDTD), Laguerre polynomials, unconditionally stable scheme.**

I. Introduction

THE finite-difference time-domain (FDTD) method has been widely used for the numerical analysis of transient electromagnetic problems because it is conditionally stable and very easy to implement [1]. Moreover, since it is a time-domain technique, one single run of simulation can provide much information over a wide-band using a broad-band excitation. However, since the FDTD method is an explicit time-marching technique, its time step size should be limited by the well-known Courant–Friedrich–Lecy (CFL) stability condition. Since the time step is dependent on the smallest length of the cell in a computational domain, this CFL condition may be too restrictive to solve problems with fine structures, such as thin material, slot, and via.

In recent times, to eliminate the CFL stability condition, the alternating-direction-implicit (ADI) method was proposed in order to formulate the implicit FDTD scheme [2]–[6]. The resulting ADI-FDTD method was found to be unconditionally stable. Therefore, one can use the larger value of the time step rather than the CFL limit. However, the larger value of the time step rather than the CFL limit results in a larger dispersion error.

In this paper, we propose an unconditionally stable solution procedure for the FDTD method for the two-dimensional (2-D) TE_z case using weighted Laguerre polynomials as temporal basis and testing functions. Since Laguerre polynomials are defined from $t=0$ to $t=+\infty$, they are suitable for a causal system [7], [8]. Laguerre polynomials of higher orders can be generated recursively and are orthogonal with respect to a weighting function in a function space defined through the inner product of two continuous functions. Using the Laguerre polynomials and the weighting function, one can construct a set of orthogonal basis functions, which we call the weighted Laguerre polynomials. Physical quantities that are functions of time can be spanned in terms of these orthogonal basis functions—weighted Laguerre polynomials. Note that the weighted Laguerre polynomials are completely convergent to zero as $t \rightarrow \infty$. Therefore, arbitrary quantities or functions spanned by these basis functions are also convergent to zero as time progresses. Using the Galerkin's method, we introduce a temporal testing procedure, which results in an implicit FDTD formulation. By applying the temporal testing procedure to the FDTD, one can eliminate the time-step limitation that is the hallmark of the explicit time-domain technique. Instead of the leapfrog procedure, we introduce a marching-on-in-order of the basis functions. Therefore, we can obtain the unknown coefficients for the basis functions from the zeroth order to the N_L^{th} order by solving recursively the FDTD with weighted Laguerre polynomials. The minimum order or number of basis functions is dependent on the time duration and the frequency–bandwidth product of the problem.

When employing the conventional FDTD method, there is no matrix inversion involved with this computation procedure. However, the proposed method produces a banded sparse system matrix and is independent of the time step. However, this method also uses the same system matrix regardless of the order of basis functions to recursively solve for the unknowns. Therefore, one can assemble this sparse system matrix only once.

Manuscript received May 30, 2002; September 18, 2002.

Y.-S. Chung is with the Department of Communication Engineering, Myongji University, Kyunggi 449-728, Korea (e-mail: ychung05@mju.ac.kr).

T. K. Sarkar is with the Department of Electrical Engineering and Computer Science, Syracuse University, Syracuse, NY 13244-1240 USA (e-mail: tksarkar@syr.edu).

B. H. Jung is with the Department of Information and Communication Engineering, Hoseo University, Chungnam 336-795, Korea (e-mail: bhjung@office.hoseo.ac.kr).

M. Salazar-Palma is with the Departamento de Señales, Sistemas y Radiocomunicaciones, Universidad Politécnica de Madrid, Madrid 28040, Spain (e-mail: Salazar@gmr.ssr.upm.es).

Digital Object Identifier 10.1109/TMTT.2003.808732

0018-9480/03$17.00 © 2003 IEEE

The paper is organized in the following manner. In Section II, the formulations of the proposed FDTD are described. In Section III, the numerical results are presented. Finally, in Section IV, we summarize some conclusions.

II. FDTD Using Weighted Laguerre Polynomials as Basis Functions

A. FDTD With the TE_z Case

With simple and lossless media, the TE_z model formulation of the time-domain Maxwell's equations is

$$\dot{E}_x = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - J_x\right) \tag{1}$$

$$\dot{E}_y = \frac{1}{\varepsilon}\left(-\frac{\partial H_z}{\partial x} - J_y\right) \tag{2}$$

$$\dot{H}_z = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \tag{3}$$

where ε is the electric permittivity and μ is the magnetic permeability. An upper dot denotes the derivative with respect to time. Using the central difference scheme both on time and space, (1)–(3) are discretized as

$$E_x|_{i,j}^{n} = E_x|_{i,j}^{n-1} + C_y^E\big|_{i,j}\left(H_z|_{i,j}^{n-1/2} - H_z|_{i,j-1}^{n-1/2}\right) - \frac{\Delta t}{\varepsilon_{i,j}} J_x|_{i,j}^{n-1/2} \tag{4}$$

$$E_y|_{i,j}^{n} = E_y|_{i,j}^{n-1} - C_x^E\big|_{i,j}\left(H_z|_{i,j}^{n-1/2} - H_z|_{i-1,j}^{n-1/2}\right) - \frac{\Delta t}{\varepsilon_{i,j}} J_y|_{i,j}^{n-1/2} \tag{5}$$

$$H_z|_{i,j}^{n+1/2} = H_z|_{i,j}^{n-1/2} + C_y^H\big|_{i,j}\left(E_x|_{i,j+1}^{n} - E_x|_{i,j}^{n}\right) - C_x^H\big|_{i,j}\left(E_y|_{i+1,j}^{n} - E_y|_{i,j}^{n}\right) \tag{6}$$

where

$$C_x^E\big|_{i,j} = \frac{\Delta t}{\varepsilon_{i,j}\Delta\bar{x}_i}, \qquad C_y^E\big|_{i,j} = \frac{\Delta t}{\varepsilon_{i,j}\Delta\bar{y}_j} \tag{7}$$

$$C_x^H\big|_{i,j} = \frac{\Delta t}{\mu_{i,j}\Delta x_i}, \qquad C_y^H\big|_{i,j} = \frac{\Delta t}{\mu_{i,j}\Delta y_j}. \tag{8}$$

For the above difference equations, (i,j) is not a real position but an array index of each field variable, as shown in Fig. 1. Fig. 1 shows the position of the electric and magnetic field vector components over the 2-D cells. Δx_i and Δy_i are the lengths of the edge where the electric fields are located. $\Delta\bar{x}_i$ and $\Delta\bar{y}_i$ are the distances between the center nodes where the magnetic fields are located.

In this paper, we use the dispersive boundary condition (DBC) as an absorbing boundary condition (ABC) [9]. The first-order DBC at $x = 0$ or X is given by

$$\left(\frac{\partial}{\partial x} \pm \frac{1}{v_1}\frac{\partial}{\partial t}\right) E_y(\mathbf{r}) = 0. \tag{9}$$

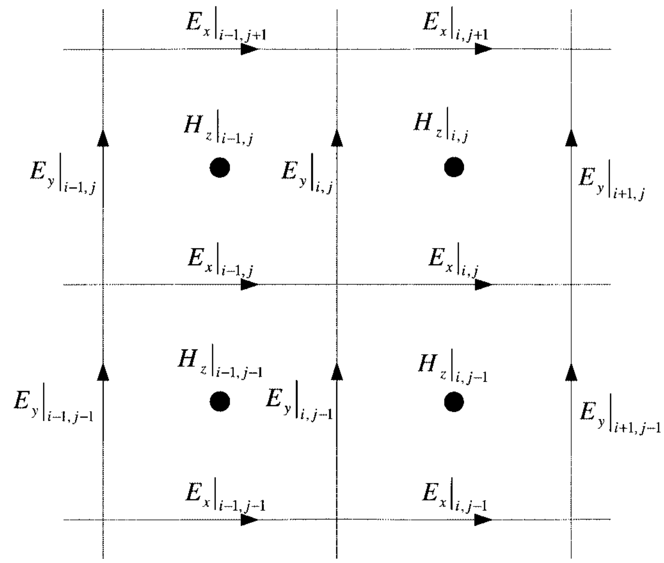


Fig. 1. Position of the transient electric and magnetic field vector components on the TE_z plane.

Using the central difference scheme and the averaging technique, the difference relation of (9) at $x = X$ (or $i = I$) can be written as

$$E_y|_{I,j}^{n} = E_y|_{I-1,j}^{n-1} - \gamma_1\left(E_y|_{I-1,j}^{n} - E_y|_{I,j}^{n-1}\right) \tag{10}$$

where

$$\gamma_1 = \frac{\Delta x - v_1\Delta t}{\Delta x + v_1\Delta t}. \tag{11}$$

Similarly, the difference relation of (9) at $x = 0$ (or $i = 1$) is

$$E_y|_{1,j}^{n} = E_y|_{2,j}^{n-1} - \gamma_1\left(E_y|_{2,j}^{n} - E_y|_{1,j}^{n-1}\right). \tag{12}$$

Since the conventional 2-D FDTD method is an explicit time-domain technique, its time step is bounded by the CFL stability condition as follows:

$$\Delta t \le \frac{1}{v_{\max}}\left(\left(\frac{1}{\Delta x_i}\right)^2 + \left(\frac{1}{\Delta y_j}\right)^2\right)^{-1/2} \tag{13}$$

where $v_{\max}$ is the maximum phase velocity. Since the time step is dependent on the smallest length of the cell in a computational domain, the CFL condition may be too restrictive to solve problems with fine structures, such as a thin material, slot, and via.

So, in order to obtain an unconditionally stable solution regardless of the time step size, we propose a new FDTD algorithm in Section II-B, which uses weighted Laguerre polynomials as an entire domain temporal basis function.

B. FDTD With Weighted Laguerre Polynomials

Consider the set of polynomials defined by

$$L_p(t) = \frac{e^t}{p!}\frac{d^p}{dt^p}\left(t^p e^{-t}\right), \qquad \text{for } p \ge 0; \quad t \ge 0. \tag{14}$$

These are Laguerre polynomials of order p that are causal, which means that they exist for $t \geq 0$. These polynomials satisfy a recursive relationship given by

$$L_0(t) = 1 \qquad L_1(t) = 1 - t \tag{15}$$

$$pL_p(t) = (2p - 1 - t)L_{p-1}(t) - (p-1)L_{p-2}(t), \quad \text{for } p \geq 2; \quad t \geq 0. \tag{16}$$

The Laguerre polynomials are orthogonal with respect to the weighting function e^{-t}, given by

$$\int_0^\infty e^{-t} L_p(t) L_q(t)\, dt = \delta_{pq} \tag{17}$$

where δ_{pq} is a Kronecker delta for $p = q$ and zero otherwise. Therefore, an orthonormal set of basis functions $\{\varphi_0, \varphi_1, \varphi_2, \ldots\}$ can be derived from (17) through the representation

$$\varphi_p(t, s) = e^{-s \cdot t/2} L_p(s \cdot t) \tag{18}$$

where $s > 0$ is a time-scale factor. Note that these functions are absolutely convergent to zero as $t \to \infty$. Hence arbitrary functions spanned by these basis functions are also absolutely convergent to zero as $t \to \infty$. These basis functions are also orthogonal with respect to the scaled time variable $\bar{t}$ as

$$\int_0^\infty \varphi_u(\bar{t}) \cdot \varphi_v(\bar{t})\, d\bar{t} = \delta_{uv} \tag{19}$$

where $\bar{t} = s \cdot t$ is the scaled time. Since the real time scale is quite small, in order to use the above basis functions properly, one should transform the real time scale using an appropriate scale factor. These orthogonal functions can approximate causal electromagnetic responses quite well. By controlling the time-scale factor s, the support provided by the expansion can be increased or decreased. Basis functions on the order of 0–4 are plotted in Fig. 2. As can be seen, the functions given by (18) are causal and convergent as $t \to \infty$.

Using these basis functions, the temporal coefficients in (1)–(3) can be expanded as

$$E_x(\mathbf{r}, t) = \sum_{p=0}^{\infty} E_x^p(\mathbf{r}) \varphi_p(\bar{t}) \tag{20}$$

$$E_y(\mathbf{r}, t) = \sum_{p=0}^{\infty} E_y^p(\mathbf{r}) \varphi_p(\bar{t}) \tag{21}$$

$$H_z(\mathbf{r}, t) = \sum_{p=0}^{\infty} H_z^p(\mathbf{r}) \varphi_p(\bar{t}). \tag{22}$$

Note that the time variable on the left-hand side is different from the one on the right-hand side. In the above equations, $\varphi_p(\bar{t})$ can be regarded as an entire domain temporal basis function. To apply (20)–(22) to (1)–(3) and (9), we express the first derivative

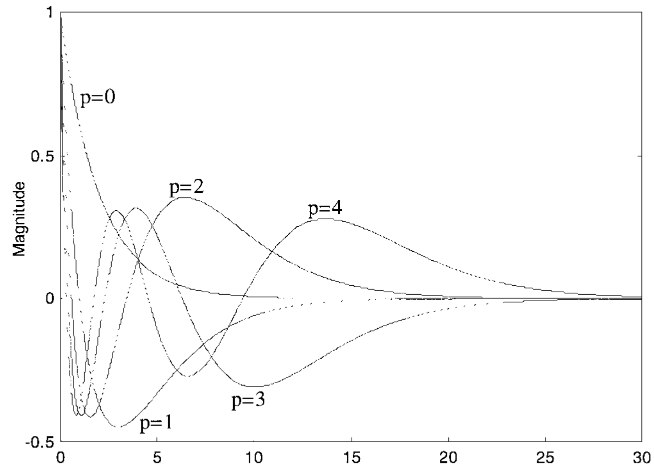


Fig. 2. Weighted Laguerre polynomials of different orders.

of the field variables with respect to time t. In [8], we can show that the first derivative of $U(\mathbf{r}, t)$ with respect to time t is

$$\dot{U}(\mathbf{r}, t) = s \sum_{p=0}^{\infty} \left(0.5U_p(\mathbf{r}) + \sum_{k=0, p>0}^{p-1} U_k(\mathbf{r}) \right) \varphi_p(\bar{t}) \tag{23}$$

where $U(\mathbf{r}, t)$ is a causal function. Inserting (20)–(22) into (1)–(3), respectively, we have

$$s \sum_{p=0}^{\infty} \left(0.5E_x^p(\mathbf{r}) + \sum_{k=0, p>0}^{p-1} E_x^k(\mathbf{r}) \right) \varphi_p(\bar{t}) = \frac{1}{\varepsilon(\mathbf{r})} \sum_{p=0}^{\infty} \frac{\partial}{\partial y} H_z^p(\mathbf{r}) \varphi_p(\bar{t}) - \frac{J_x(\mathbf{r}, t)}{\varepsilon(\mathbf{r})} \tag{24}$$

$$s \sum_{p=0}^{\infty} \left(0.5E_y^p(\mathbf{r}) + \sum_{k=0, p>0}^{p-1} E_y^k(\mathbf{r}) \right) \varphi_p(\bar{t}) = -\frac{1}{\varepsilon(\mathbf{r})} \sum_{p=0}^{\infty} \frac{\partial}{\partial x} H_z^p(\mathbf{r}) \varphi_p(\bar{t}) - \frac{J_y(\mathbf{r}, t)}{\varepsilon(\mathbf{r})} \tag{25}$$

$$s \sum_{p=0}^{\infty} \left(0.5H_z^p(\mathbf{r}) + \sum_{k=0, p>0}^{p-1} H_z^k(\mathbf{r}) \right) \varphi_p(\bar{t}) = \frac{1}{\mu(\mathbf{r})} \sum_{p=0}^{\infty} \left[\frac{\partial}{\partial y} E_x^p(\mathbf{r}) - \frac{\partial}{\partial x} E_y^p(\mathbf{r}) \right] \varphi_p(\bar{t}). \tag{26}$$

To eliminate the time-dependent terms $\varphi_p(\bar{t})$, we introduce a temporal Galerkin's testing procedure of (24)–(26) by using the orthogonal property of the weighted Laguerre functions. We multiply both sides of (24)–(26) by $\varphi_q(\bar{t})$ and integrate over $\bar{t} = [0, \infty)$. Then, we get

$$s \left(0.5E_x^q(\mathbf{r}) + \sum_{k=0, q>0}^{q-1} E_x^k(\mathbf{r}) \right) = \frac{1}{\varepsilon(\mathbf{r})} \frac{\partial}{\partial y} H_z^q(\mathbf{r}) - \frac{J_x^q(\mathbf{r})}{\varepsilon(\mathbf{r})} \tag{27}$$

$$s\left(0.5E_y^q(\mathbf{r}) + \sum_{k=0,q>0}^{q-1} E_y^k(\mathbf{r})\right) = -\frac{1}{\varepsilon(\mathbf{r})}\frac{\partial}{\partial x}H_z^p(\mathbf{r}) - \frac{J_y^q(\mathbf{r})}{\varepsilon(\mathbf{r})} \tag{28}$$

$$s\left(0.5H_z^q(\mathbf{r}) + \sum_{k=0,q>0}^{q-1} H_z^k(\mathbf{r})\right) = \frac{1}{\mu(\mathbf{r})} \times \left(\frac{\partial}{\partial y}E_x^q(\mathbf{r}) - \frac{\partial}{\partial x}E_y^q(\mathbf{r})\right) \tag{29}$$

where

$$J_x^q(\mathbf{r}) = \int_0^{T_f} J_x(\mathbf{r},t)\,\varphi_q(\bar{t})\,d\bar{t} \tag{30}$$

$$J_y^q(\mathbf{r}) = \int_0^{T_f} J_y(\mathbf{r},t)\,\varphi_q(\bar{t})\,d\bar{t}. \tag{31}$$

The upper limit of infinity can be replaced by a finite time interval T_f. This interval is chosen in such a way that the waveforms of interest have practically decayed to zero. This is the temporal testing procedure with respect to the basis function of order q, $\varphi_q(\bar{t})$. Rewriting (27)–(29) in a matrix form, we have

$$E_x^q|_{i,j} = \bar{C}_y^E\big|_{i,j}\left(H_z^q|_{i,j} - H_z^q|_{i,j-1}\right) - \frac{2}{s\varepsilon}J_x^q|_{i,j} - 2\sum_{k=0}^{q-1}E_x^k\big|_{i,j} \tag{32}$$

$$E_y^q\big|_{i,j} = -\bar{C}_x^E\big|_{i,j}\left(H_z^q|_{i,j} - H_z^q|_{i-1,j}\right) - \frac{2}{s\varepsilon}J_y^q\big|_{i,j} - 2\sum_{k=0}^{q-1}E_y^k\big|_{i,j} \tag{33}$$

$$H_z^q|_{i,j} = -\bar{C}_x^H\big|_{i,j}\left(E_y^q\big|_{i+1,j} - E_y^q\big|_{i,j}\right) + \bar{C}_y^H\big|_{i,j}\left(E_x^q|_{i,j+1} - E_x^q|_{i,j}\right) - 2\sum_{k=0}^{q-1}H_z^k\big|_{i,j} \tag{34}$$

where

$$\bar{C}_x^E\big|_{i,j} = \frac{2}{s\varepsilon_{i,j}\Delta\bar{x}_i} \tag{35}$$

$$\bar{C}_y^E\big|_{i,j} = \frac{2}{s\varepsilon_{i,j}\Delta\bar{y}_j} \tag{36}$$

$$\bar{C}_x^H\big|_{i,j} = \frac{2}{s\mu_{i,j}\Delta x_i} \tag{37}$$

$$\bar{C}_y^H\big|_{i,j} = \frac{2}{s\mu_{i,j}\Delta y_j}. \tag{38}$$

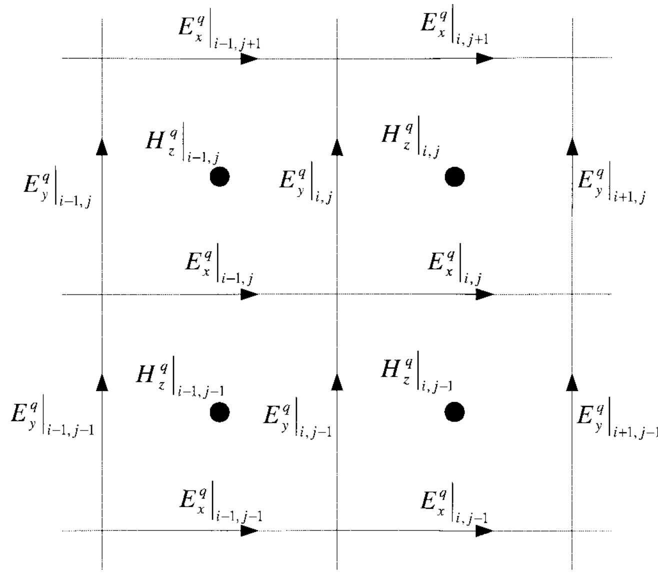


Fig. 3. Position of the electric and magnetic fields of the order q on the TE_z plane.

Contrary to the conventional FDTD difference, (4)–(6) and (32)–(34) have an implicit relation. Therefore, inserting (34) into (32) and (33), and rewriting (32) and (33), we have

$$\begin{aligned} &-\bar{C}_y^H\big|_{i,j-1}E_x^q|_{i,j-1} + \left(\frac{1}{\bar{C}_y^E\big|_{i,j}} + \bar{C}_y^H\big|_{i,j} + \bar{C}_y^H\big|_{i,j-1}\right) \times E_x^q|_{i,j} - \bar{C}_y^H\big|_{i,j-1}E_x^q|_{i,j+1} \\ &\quad + \bar{C}_x^H\big|_{i,j-1}E_y^q\big|_{i-1,j} + \bar{C}_x^H\big|_{i,j-1}E_y^q\big|_{i-1,j-1} - \bar{C}_x^H\big|_{i,j}E_y^q\big|_{i,j} + \bar{C}_x^H\big|_{i,j}E_y^q\big|_{i+1,j} \\ &= -\Delta y_j\,J_x^q|_{i,j} - \frac{2}{\bar{C}_y^E\big|_{i,j}}\sum_{k=0}^{q-1}E_x^k\big|_{i,j} - 2\sum_{k=0}^{q-1}\left(H_z^k\big|_{i,j} - H_z^k\big|_{i,j-1}\right) \end{aligned} \tag{39}$$

$$\begin{aligned} &\bar{C}_y^H\big|_{i,j}E_x^q|_{i,j+1} - \bar{C}_y^H\big|_{i-1,j}E_x^q|_{i-1,j+1} - \bar{C}_y^H\big|_{i,j}E_x^q|_{i,j} + \bar{C}_y^H\big|_{i,j}E_x^q|_{i-1,j} - \bar{C}_x^H\big|_{i-1,j}E_y^q\big|_{i-1,j} \\ &\quad + \left(\frac{1}{\bar{C}_x^E\big|_{i,j}} + \bar{C}_x^H\big|_{i,j} + \bar{C}_x^H\big|_{i-1,j}\right)E_y^q\big|_{i,j} - \bar{C}_x^H\big|_{i,j}E_y^q\big|_{i+1,j} \\ &= -\Delta x_i\,J_y^q\big|_{i,j} - \frac{2}{\bar{C}_x^E\big|_{i,j}}\sum_{k=0}^{q-1}E_y^k\big|_{i,j} - 2\sum_{k=0}^{q-1}\left(H_z^k\big|_{i,j} - H_z^k\big|_{i-1,j}\right). \end{aligned} \tag{40}$$

From (39) and (40), we can find that each electric field variable has the relationship with the adjacent six electric fields, as shown in Fig. 3. Note that, in (39) and (40), the magnetic fields are known because their orders are lower than those of the electric fields. Therefore, each row has seven nonzero terms. Rewriting (39) and (40) as a matrix equation, we have

$$[A]\{E^q\} = \{J^q\} + \{\beta^{q-1}\}, \qquad q = 0, 1, 2, \ldots \tag{41}$$

In (42), $\{E^q\} = \{E_x^q, E_y^q\}^T$, and $\{J^q\} = \{J_x^q, J_y^q\}^T$ is the term due to an incident electric current expressed in (30) and (31). $\{\beta^{q-1}\}$ is the summation term from the order 0 to $q-1$. However, in the case of the boundary edges (perfect electric conductor (PEC) and ABC), we need a different procedure. First, in the case of the PEC boundary condition, all terms of its rows and columns except its diagonal term should be replaced with zero. Also, its row term of the forcing vectors $\{J^q\}$ and $\{\beta^{q-1}\}$ should be zero.

In the case of the first-order DBC, inserting (23) into (9) and applying the temporal testing procedure with $\varphi_q(\bar{t})$, we can eliminate the time derivative. Then, at $x = X$, we obtain

$$\frac{\partial}{\partial x}E_y^q(\mathbf{r}) + \frac{s}{v_1}\left(\frac{E_y^q(\mathbf{r})}{2} + \sum_{k=0}^{q-1} E_y^k(\mathbf{r})\right) = 0. \tag{42}$$

Using the averaging technique and the central difference scheme at an auxiliary grid point $(I-1/2, j)$, we can transform (42) into a difference equation

$$E_y^p\big|_{I-1/2,j} = \frac{E_y^p\big|_{I,j} + E_y^p\big|_{I-1,j}}{2} \tag{43}$$

$$\frac{\partial}{\partial x} E_y^p\big|_{I-1/2,j} = \frac{E_y^p\big|_{I,j} - E_y^p\big|_{I-1,j}}{\Delta x} \tag{44}$$

$$\left(\frac{s}{4v_1} + \frac{1}{\Delta x}\right) E_y^q\big|_{I,j} + \left(\frac{s}{4v_1} - \frac{1}{\Delta x}\right) E_y^q\big|_{I-1,j} = -\frac{s}{2v_1}\sum_{k=0}^{q-1}\left(E_y^k\big|_{I,j} + E_y^k\big|_{I-1,j}\right). \tag{45}$$

Similarly, at $x = 0$, we have the ABC difference equation as follows:

$$\left(\frac{s}{4v_1} + \frac{1}{\Delta x}\right) E_y^q\big|_{1,j} + \left(\frac{s}{4v_1} - \frac{1}{\Delta x}\right) E_y^q\big|_{2,j} = -\frac{s}{2v_1}\sum_{k=0}^{q-1}\left(E_y^k\big|_{2,j} + E_y^k\big|_{1,j}\right). \tag{46}$$

Inserting (45), (46), and the PEC boundary condition into (41), we have a modified matrix equation as follows:

$$\left[\tilde{A}\right]\{E^q\} = \left\{\tilde{J}^q\right\} + \left\{\tilde{\beta}^{q-1}\right\}, \qquad q = 0, 1, 2, \ldots \tag{47}$$

Contrary to the conventional FDTD method, the proposed method has an implicit relationship between the field variables, which results in a sparse system matrix $\left[\tilde{A}\right]$. However, one can observe that the system matrix $\left[\tilde{A}\right]$ is independent of the order q of the temporal testing function $\varphi_q(\bar{t})$. Note that the stability is no longer affected by the time step size. In our method, the time step is used only to calculate the Laguerre coefficients due to the excitation in (30) and (31), at the start of the computations. Therefore, one can choose a small value of Δt to evaluate (30) and (31) accurately, which does not increase the computing time.

In (47), since $\left\{\tilde{J}^q\right\}$ is a known vector, if we know the coefficients for the electric fields from order 0 to $q-1$, one can solve (47). Inserting $q = 0$ into (47) and rewriting (47), we obtain

$$\left[\tilde{A}\right]\{E^0\} = \left\{\tilde{J}^0\right\} \tag{48}$$

because $\left\{\tilde{\beta}^{-1}\right\} = 0$. Since $\left\{\tilde{J}^0\right\}$ is a known vector, we can obtain $\{E^0\}$ easily by solving the matrix (48).

Since at each recursion the proposed method uses the same system matrix first, one can perform the lower–upper (LU) decomposition of $\left[\tilde{A}\right]$ only once at the beginning of the computation step. One can then solve (47) by using the back-substitution routine repeatedly. The magnetic fields can be obtained from (34).

In Section II-C, we explain up to what order of the temporal basis functions one should consider. This parameter is related to the accuracy of the solution regardless of the stability condition.

C. Choice of the Number of Temporal Basis Functions

It is assumed that the signal that we are interested in characterizing is practically band-limited up to a frequency B. In addition, we are also interested in generating the same signal in the time domain up to the time duration T_f. Then, we represent the real-time signal $P(t)$ by a Fourier series

$$P(t) = \sum_u C_u e^{ju\omega_0 t} \tag{49}$$

where $\omega_0 = 2\pi/T_f$. Since $P(t)$ is real, $C_u^* = C_{-u}$ where $*$ means conjugate transpose. If $P(t)$ is band-limited to B Hertz, then the value of u can be fixed by

$$-B \le \frac{u}{T_f} \le B. \tag{50}$$

Therefore, we have

$$P(t) = \sum_{u=-BT_f}^{BT_f} C_u e^{ju\omega_0 t}. \tag{51}$$

In (51), there are $2BT_f + 1$ terms in the expansion of $P(t)$. Hence, the minimum number of temporal basis functions is

$$N_L = 2BT_f + 1. \tag{52}$$

In order to obtain an accurate solution, therefore, one should solve (47) recursively at least N_L times. Therefore, if we want to observe the transient response at a spatial location due to an incident field of bandwidth $2B$, then we need at least $2BT_f + 1$ terms of the Laguerre series to completely characterize that temporal waveform of duration T_f and bandwidth $2B$, irrespective of its shape.

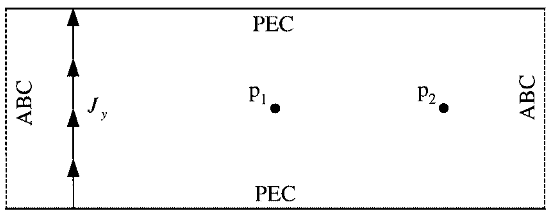
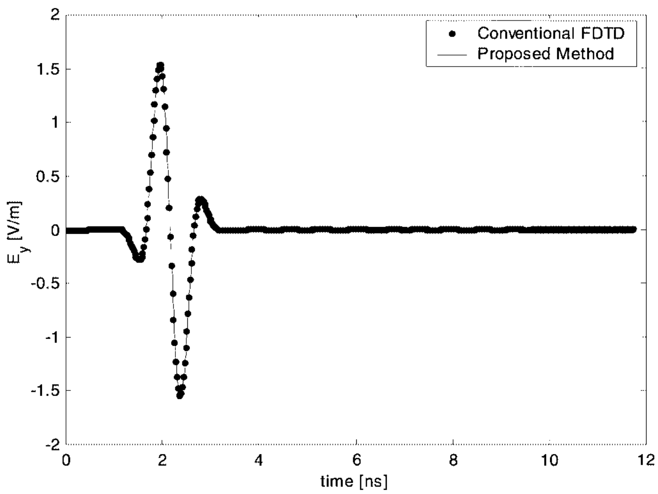
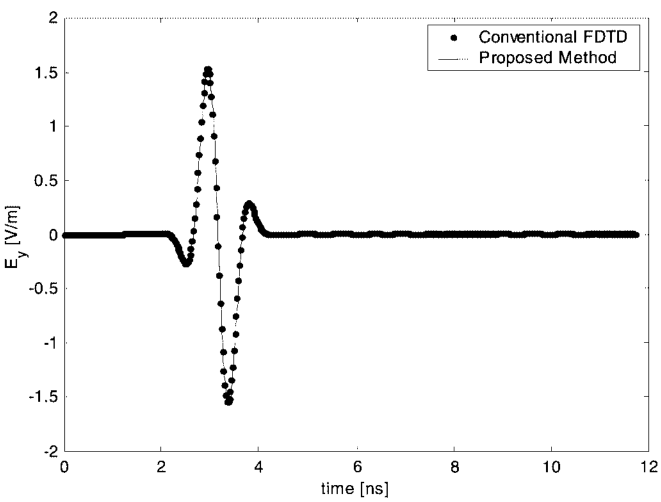


Fig. 4. 2-D parallel plate waveguide with the length 1.0 m and the width 0.1 m.



(a)



(b)

Fig. 5. Transient electric fields of the y component (a) at p_1 and (b) at p_2.

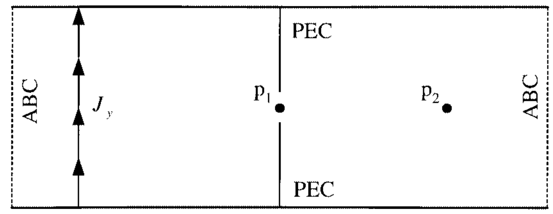
D. Calculation of Electric and Magnetic Fields

By solving (47) recursively, we obtain the coefficients of each temporal basis function, which are the expansion coefficients of the electric and magnetic fields. From (20)–(22), we obtain

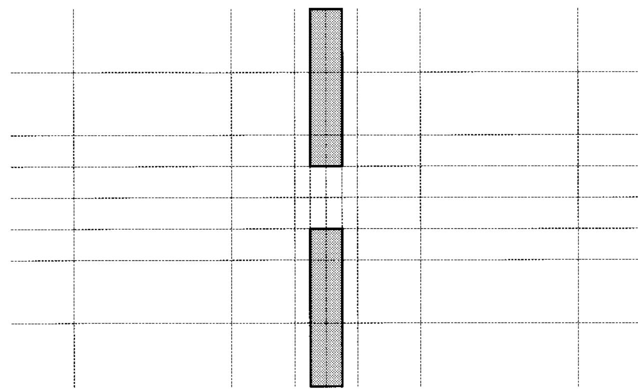
$$E_x|_{i,j}(t) = \sum_{p=0}^{N_L} E_x^p|_{i,j}\,\varphi_p(\bar{t}) \tag{53}$$

$$E_y|_{i,j}(t) = \sum_{p=0}^{N_L} E_y^p\big|_{i,j}\,\varphi_p(\bar{t}) \tag{54}$$

$$H_z|_{i,j}(t) = \sum_{p=0}^{N_L} H_z^p|_{i,j}\,\varphi_p(\bar{t}). \tag{55}$$



(a)



(b)

Fig. 6. 2-D parallel plate waveguide with the thin PEC slot of the thickness 1.0 μm and the distance 1.0 cm. (a) Computational domain. (b) Graded discretization in the vicinity of the slot.

In (18), one can see that $\lim_{\bar{t}\to\infty} \varphi_p(\bar{t}) = 0$. So, we can observe that the electric and magnetic fields obtained from the above equations are unconditionally stable because they are spanned by a set of absolutely convergent basis functions as $t \to \infty$.

III. Numerical Examples

In this section, 2-D parallel plate waveguides for the TE_z case are tested to validate our method. In this paper, we use the following sinusoidally modulated Gaussian pulse as an input electric current profile:

$$J_y(t) = \exp\left(-\left(\frac{t-T_c}{T_d}\right)^2\right)\sin\left(2\pi f_c\,(t-T_c)\right) \tag{56}$$

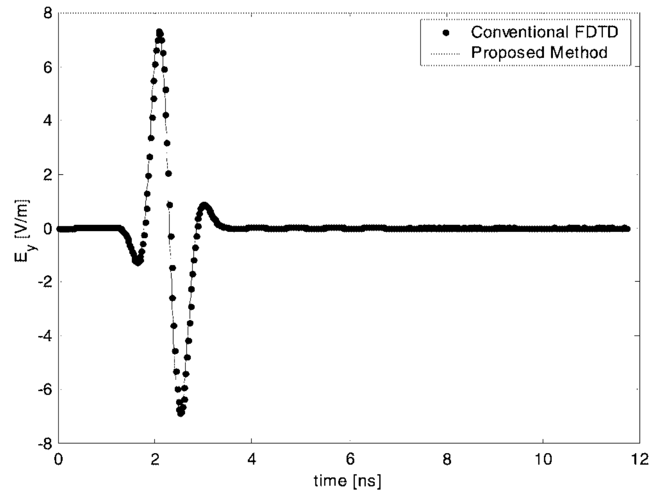
where

$$T_d = \frac{1}{2f_c}$$
$$T_c = 3T_d. \tag{57}$$

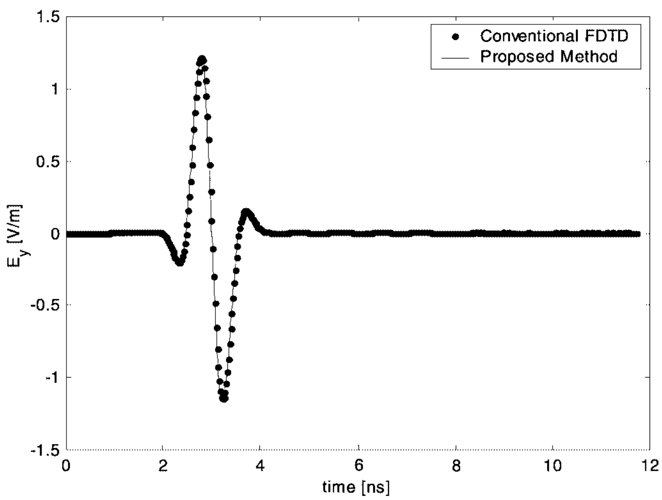
In this paper, we use $f_c = 1$ GHz, and we choose $T_f = 11.71$ ns and $B = 5$ GHz. Inserting T_f and B into (52), we can evaluate the number of the weighted Laguerre polynomial functions, and we choose $N_L = 150$. Also, the time-scale factor is $s = 6.07 \times 10^{10}$.

As the first example, we consider a 2-D parallel plate waveguide as shown in Fig. 4. There are 100 and 10 uniform subdivisions along the x and y directions, respectively. The cell size is 0.01 m × 0.01 m. The y component of the electric current is located on the excitation line, and there are two measurement points, p_1 and p_2. The first-order dispersive boundary conditions are set at the x-directional terminals of the domain, and we use $v_1 = c_0$.

Fig. 5 shows the y components of the electric fields at p_1 and p_2. The agreement between the conventional FDTD method and the proposed method is very good. The CFL stability condition



(a)



(b)

Fig. 7. Transient electric fields of the y component (a) at p_1 and (b) p_2.

of this model is $\Delta t < 23.57$ ps. The time step size for the proposed method is $\Delta t = 10.0$ ps in order to accurately evaluate (30) and (31), which hardly increases the computing time.

As the next example, we consider a 2-D parallel plate waveguide with a thin PEC slot, as shown in Fig. 6(a). In order to model this thin conductor plate of width 1.0 μm, we use the graded mesh as shown in Fig. 6(b). A very small cell is placed around the slot. In the PEC plate model, the PEC plate is divided into two cells, and the minimum cell size is 0.5 μm$\times$ 0.005 m. There are 120 and 12 subdivisions along the x and y directions, respectively. The CFL stability condition of this model is $\Delta t < 3.333$ fs. The time step size chosen for the proposed method is $\Delta t = 10.0$ ps to calculate the Laguerre coefficients of the excitation pulse, which is small enough to evaluate (30) and (31). Fig. 7 shows the y components of the electric fields at p_1 and p_2, respectively. The agreement between the conventional FDTD method and the proposed method is very good. Table I represents the required computational resource and the computing time for the numerical simulations. Since the proposed method requires a smaller number of iterations than those of the conventional FDTD method, the CPU time for the proposed method can be reduced to about 1.26% of the original

TABLE I
COMPARISON OF THE COMPUTATIONAL EFFORTS FOR THE PARALLEL PLATE WAVEGUIDE WITH A THIN SLOT IN A CONDUCTOR

	Δt	No. of iterations	CPU time
FDTD	3.33 fs	3570699	214 s
Our Method	10 ps	151	2.7 s

FDTD method while maintaining the high accuracy. Therefore, by not using the CFL stability conditions and completely eliminating the time variable from the computations of the updating of the electric and the magnetic fields, the computation of the proposed method is less than the computation time of a conventional FDTD method by at least two orders of magnitude. All calculations in this paper have been performed on an Intel Pentium IV 2.2-GHz machine.

IV. CONCLUSION

An unconditionally stable solution for the FDTD algorithm has been proposed for the 2-D TE_z case with fine structures. To model a very thin conductor slot, graded discretization modeling has been employed to provide more flexibility. We utilize a marching-on-in-order method to solve the FDTD with weighted Laguerre polynomials. As entire domain temporal basis and testing functions, the advantages of using the weighted Laguerre polynomials are: 1) it guarantees an unconditional stability; 2) the solution is independent of the time discretization; 3) the temporal derivatives can be treated analytically; and 4) most importantly, from a computational standpoint, the spatial and the temporal variables can be separated, resulting in an efficient and accurate solution. Transient fields obtained by the present method are unconditionally stable regardless of the time step size. Moreover, the agreement between the results obtained using the proposed method and the conventional FDTD method is very good. Currently, it is being extended to three-dimensional problems.

REFERENCES

[1] A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*. Boston, MA: Artech House, 2000.
[2] T. Namiki, “A new FDTD algorithm based on alternating-direction implicit method,” *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 2003–2007, Oct. 1999.
[3] F. Zeng, Z. Chen, and J. Zhang, “Toward the development of a three-dimensional unconditionally stable finite-difference time-domain method,” *IEEE Trans. Microwave Theory Tech.*, vol. 48, pp. 1550–1558, Sept. 2000.
[4] T. Namiki, “3-D ADI-FDTD method—Unconditionally stable time-domain algorithm for solving full vector Maxwell’s equations,” *IEEE Trans. Microwave Theory Tech.*, vol. 48, pp. 1743–1748, Oct. 2000.
[5] T. Namiki and K. Ito, “Numerical simulation of microstrip resonators and filters using the ADI-FDTD method,” *IEEE Trans. Microwave Theory Tech.*, vol. 49, pp. 665–670, Apr. 2001.
[6] ——, “Numerical simulation using ADI-FDTD method to estimate shielding effectiveness of thin conductive encloses,” *IEEE Trans. Microwave Theory Tech.*, vol. 49, pp. 1060–1066, June 2001.
[7] A. D. Poularikas, *The Transforms and Applications Handbook*. Piscataway, NJ: IEEE Press, 1996.
[8] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. New York: Academic, 1980.

[9] Z. Bi, K. Wu, C. Wu, and J. Litva, "A dispersive boundary condition for microstrip component analysis using the FD-TD method," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 774–777, Apr. 1992.

[10] B. Z. Wang, "Enhanced thin-slot formalism for the FDTD analysis of thin-slot penetration," *IEEE Microwave Guided Wave Lett.*, vol. 5, pp. 142–143, May 1995.

Young-Seek Chung (A'01–M'01) received the B.S., M.S., and Ph.D. degrees in electrical engineering from the Seoul National University, Seoul, Korea, in 1989, 1991, and 2000, respectively.

From 1991 to 1996, he was with the Living System Laboratory, LG Electronics. From 1998 to 2000, he was a Teaching Assistant of electrical engineering with the Seoul National University. From 2001 to 2002, he was with Syracuse University, Syracuse, NY. Since 2003, he has been a faculty member with the Department of Communication Engineering, Myongji University, Kyunggi, Korea. His current interests are numerical analysis and inverse scattering using time-domain techniques.

Tapan K. Sarkar (S'69–M'76–SM'81–F'92) received the B.Tech. degree from the Indian Institute of Technology, Kharagpur, in 1969, the M.Sc.E. degree from the University of New Brunswick, Fredericton, NB, Canada, in 1971, and the M.S. and Ph.D. degrees from Syracuse University, Syracuse, NY, in 1975.

From 1975 to 1976, he was with the TACO Division of the General Instruments Corporation. He was with the Rochester Institute of Technology, Rochester, NY, from 1976 to 1985. He was a Research Fellow at the Gordon McKay Laboratory, Harvard University, Cambridge, MA, from 1977 to 1978. He is now a Professor in the Department of Electrical and Computer Engineering, Syracuse University. His current research interests deal with numerical solutions of operator equations arising in electromagnetics and signal processing with application to system design. He obtained one of the "best solution" awards in May 1977 at the Rome Air Development Center (RADC) Spectral Estimation Workshop. He has authored or coauthored more than 210 journal articles and numerous conference papers and has written 28 chapters in books and ten books, including his most recent, *Iterative and Self Adaptive Finite-Elements in Electromagnetic Modeling* (Boston, MA: Artech House, 1998).

Dr. Sarkar is a Registered Professional Engineer in the State of New York. He received the Best Paper Award of the IEEE Transactions on Electromagnetic Compatibility in 1979 and in the 1997 National Radar Conference. He received the College of Engineering Research Award in 1996 and the Chancellor's Citation for Excellence in Research in 1998 at Syracuse University. He was an Associate Editor for feature articles of the IEEE Antennas and Propagation Society Newsletter, and he was the Technical Program Chairman for the 1988 IEEE Antennas and Propagation Society International Symposium and URSI Radio Science Meeting. He is on the editorial board of *Journal of Electromagnetic Waves and Applications* and *Microwave and Optical Technology Letters*. He has been appointed a U.S. Research Council Representative to many URSI General Assemblies. He was the Chairman of the Intercommission Working Group of International URSI on Time Domain Metrology (1990–1996). He is a member of Sigma Xi and International Union of Radio Science Commissions A and B. He received the title Docteur Honoris Causa from Universite Blaise Pascal, Clermont Ferrand, France in 1998 and the medal of the city of Clermont Ferrand, France, in 2000.

Baek Ho Jung was born in Korea, in 1961. He received the B.S., M.S., and Ph.D. degrees in electronic and electrical engineering from the Kyungpook National University, Taegu, Korea, in 1986, 1989, and 1997, respectively.

From 1989 to 1994, he was a Researcher with the Agency for Defense Development. Since 1997, he has been a Lecturer and is currently an Assistant Professor with the Department of Information and Communication Engineering, Hoseo University, Asan, Korea. From 2001 to 2002, he was with Syracuse University, Syracuse, NY, as a Visiting Scholar. His current interests are computational electromagnetics and wave propagation.

Magdalena Salazar-Palma (M'89–SM'01) was born in Granada, Spain. She received the Ingeniero de Telecomunicación and Ph.D. degrees from the Universidad Politécnica de Madrid, Madrid, Spain.

She is currently a Profesor Titular with the Departamento de Señales, Sistemas y Radiocomunicaciones, Escuela Técnica Superior de Ingenieros de Telecomunicación, Universidad Politécnica de Madrid. She has taught courses on electromagnetic-field theory, microwave and antenna theory, circuit networks and filter theory, analog and digital communication systems theory, numerical methods for electromagnetic-field problems, as well as related laboratories. She has developed her research within the Grupo de Microondas y Radar in the areas of electromagnetic-field theory, computational and numerical methods for microwave structures, passive components, and antenna analysis; design, simulation, optimization, implementation, and measurements of hybrid and monolithic microwave integrated circuits; and network and filter theory and design. On numerous occasions, she has been a Visiting Professor with the Electrical Engineering and Computer Science, Syracuse University, Syracuse, NY. She authored one book and has authored or coauthored a total of 15 contributions for chapters and papers in books published internationally, 25 papers in international journals, and 113 papers in international conferences, symposiums, and workshops, plus a number of national publications and reports. She has acted several times as Topical Editor for the disk of references of the triennial *Review of Radio Science*. She is a member of the Editorial Board of three scientific journals.

Dr. Salazar-Palma is a Registered Engineer in Spain. She has served as vice-chairman and chairman of the IEEE MTT-S (Microwave Theory and Techniques Society)/AP-S (Antennas and Propagation Society) Spain joint chapter and chairman of the IEEE Spain Section. She is a member of the IEEE Region 8 Nominations and Appointments Committee, IEEE Ethics and Member Conduct Committee, and the current chair of the IEEE Women in Engineering Committee (WIEC). She is the acting liaison between the IEEE Regional Activities Board and the IEEE WIEC. She has delivered a number of invited presentations, lectures, and seminars. She has lectured on several short courses, some of them in the frame of European Community Programs. She has participated in 42 projects and contracts, which were financially supported by international, European, and national institutions and companies. She has been a member of the Technical Program Committee of several international symposiums and has acted as reviewer for different international scientific journals, symposiums, and editorial companies. She has assisted the Comisión Interministerial de Ciencia y Tecnología in the evaluation of projects. She has also served on several evaluation panels of the Commission of the European Communities. She was the recipient of two individual research awards presented by national institutions.